Rippled area formed by surface plasmon polaritons upon femtosecond laser double-pulse irradiation of silicon

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Abstract: The formation of near-wavelength laser-induced periodic surface structures (LIPSS) on silicon upon irradiation with sequences of Ti:sapphire femtosecond laser pulse pairs (pulse duration 150 fs, central wavelength 800 nm) is studied theoretically. For this purpose, the nonlinear generation of conduction band electrons in silicon and their relaxation is numerically calculated using a two-temperature model approach including intrapulse changes of optical properties, transport, diffusion and recombination effects. Following the idea that surface plasmon polaritons (SPP) can be excited when the material turns from semiconducting to metallic state, the “SPP active area” is calculated as function of fluence and double-pulse delay up to several picoseconds and compared to the experimentally observed rippled surface areas. Evidence is presented that multi-photon absorption explains the large increase of the rippled area for temporally overlapping pulses. For longer double-pulse delays, relevant relaxation processes are identified. The results demonstrate that femtosecond LIPSS on silicon are caused by the excitation of SPP and can be controlled by temporal pulse shaping.

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References


spatial periods significantly smaller than the irradiation wavelength ($\lambda$) on a rough surface [6,7]. In contrast, on dielectrics, often high-spatial-frequency LIPSS (HSFL) with period $\lambda/2$ have been observed [2,8]. The origin of the HSFL is controversially discussed in the literature [1,8,9].

On silicon, predominantly LSFL were observed after low repetition rate ($\leq 1$ kHz) Ti:Sapphire femtosecond laser pulse irradiations in air environment [2,10–14]. Their orientation is perpendicular to the laser beam polarization and the periods typically range between $\sim 0.6\lambda$ and $\lambda$, depending on the degree of material excitation [13,15], and the number of laser pulses per spot [14]. Several authors have suggested that these structures are caused by excitation of surface plasmon polaritons (SPP) at the air-silicon interface when the material turns from a semiconducting into a metallic state [3,13,16]. The interference between the electromagnetic field of the SPP and the incident laser pulse leads to a spatially modulated deposition of optical energy to the electronic system of the material. After coupling to the lattice system [17] and subsequent ablation processes, this results in a periodically corrugated surface topography [18]. The corresponding LSFL modulation depths are typically of the order of the
optical penetration depth in the laser-excited material (∼ 100 nm) [19, 20], which is consistent with the plasmonic scenario acting at the air-silicon interface.

Two different experimental approaches were reported to study the dynamics of the LSFL formation upon irradiation of silicon by ultrashort laser pulses. Recently, in a femtosecond XUV scattering (pump-probe) experiment after single pulse irradiation of a thin silicon film, evidence for intrapulse energy deposition at specific spatial frequencies (associated with LSFL) was presented [21]. In a second approach [22, 23], multiple equal energy double-fs-pulse sequences of variable delay were used to elucidate the importance of inter- and intra-pulse feedback phenomena [24]. For silicon, the LSFL spatial period does not significantly depend on the double-pulse delay $\Delta t$ [23], while the LSFL rippled area strongly decreased with increasing delays up to several picoseconds [22]. In the latter work, two characteristic exponential decay times of $\sim 0.15$ ps and $\sim 11$ ps were found.

In this work, we extend a recently developed model [15, 25] into a double-pulse excitation scheme in order to calculate the carrier dynamics and to investigate the involvement of SPP excitation in LSFL formation on crystalline silicon. The method allows to quantify the SPP active area for different delays which is compared with experimental results. The importance of carrier generation and relaxation processes in the conduction band is demonstrated.

2. Theoretical model

Upon high-intensity laser irradiation of crystalline silicon with femtosecond laser pulses, electrons can be promoted from the valence band (VB) to the conduction band (CB) via linear and nonlinear optical absorption effects. Once a sufficiently large amount of electrons is promoted, the optical properties (e.g. the complex dielectric permittivity $\varepsilon_{Si}^*$) are affected. In presence of such a dense electron-hole plasma at CB electron densities $N_e \gtrsim 4 \times 10^{21}$ cm$^{-3}$, the silicon transiently turns from a semiconducting to a metal-like high-reflective state [13, 26, 27].

Surface plasmon polaritons can be excited at the air / excited silicon interface when the real part of the dielectric permittivity falls below minus one [28], as expressed by :

\[ \Re e [\varepsilon_{Si}^*] < -1 \]  

This occurs when a threshold carrier density $N_{spp}^{th}$ is exceeded. As usually Gaussian beam profiles are used, the spatial distribution of laser-induced electrons in the CB along with the threshold defines an “SPP active area” ($A_{spp}$) in which surface plasmon polaritons can be excited. This idea is illustrated in Fig. 1, where the hatched circular zone represents $A_{spp}$. Using numerical methods, the temporal evolution of the $A_{spp}$ can be evaluated for different double-pulse delays.

2.1. Description of optical properties

The dielectric permittivity of laser-excited silicon $\varepsilon_{Si}^*$ with carrier densities $N_e$ up to $10^{22}$ cm$^{-3}$ can be described using a Drude model given by [27]

\[ \varepsilon_{Si}^*(N_e) = \varepsilon_{Si} - \frac{\omega_p^2(N_e)}{\omega^2 (1 + i \nu / \omega)}, \]  

where $\varepsilon_{Si} = 13.64 + 0.048i$ is the dielectric constant of the non-excited silicon at $\lambda = 800$ nm wavelength [29], $\omega_p$ is the plasma frequency defined as $\omega_p = \sqrt{N_e e^2 / (m_e \varepsilon_0)}$ [$e$: electron charge, $m_e^* = 0.18 \times m_e$: effective optical mass, $m_e$: free electron mass, $\varepsilon_0$: dielectric permittivity of the vacuum], $\nu^{-1} = 1.1$ fs is the carrier collision time in laser-excited silicon [27], and $\omega$ is the laser angular frequency. Band filling and band gap renormalization effects are neglected here.
In order to consider intrapulse changes of the surface reflectivity of silicon, a multilayer model was used for normal incident radiation. The laser-excited region is considered as a layered structure where each layer $j$ has its own optical properties (related to the local free-carrier density), as defined in Eq. (2). The fraction of the laser wave amplitude reflected at the interface between layers $j$ and $k$ is calculated by the recurrence formula [30, 31]

$$r_{j,k} = \frac{r_{j,j+1} + r_{j+1,k}e^{2\phi_{j+1}}}{1 + r_{j,j+1}r_{j+1,k}e^{2\phi_{j+1}}}$$

(3)

where $r_{j,j+1}$ is given by Fresnel equations [32]. $\phi_{j+1} = \frac{2\pi h_{j+1}}{\lambda} \times \sqrt{\varepsilon_{j+1}}$ is the phase shift induced by the $j+1$ layer of thickness $h_{j+1}$. The final value of the surface reflectivity $R$ is determined by $R = |r_{0,N}|^2$ where 0 corresponds to air and $N$ is a layer located at a depth significantly larger than the optical penetration depth.

2.2. Free-carrier dynamics

The electron density in the conduction band $N_e$ was calculated considering one- and two-photon absorption, carrier diffusion and Auger recombination by solving the following partial differential equation [15]

$$\frac{\partial N_e}{\partial t} = \nabla (k_BT_e\mu_e\nabla N_e) + G_e - R_e - G_e - R_e.$$  

(4)

$G_e = \sigma_1 I_x + \sigma_2 I_y^2$ describes the electron generation rate using the one-photon ($\sigma_1$) and two-photon ($\sigma_2$) absorption coefficients listed in Table 1. $I \equiv I(t,z)$ is the temporal intensity profile within the sample upon laser excitation. $R_e = \frac{N_e}{\tau_{AR} + (C_{AR}N_e)^2}$ describes the loss rate of CB electrons by Auger recombination [25], where $\tau_{AR} = 6$ ps is the minimal recombination time due to plasma screening [33,34] and $C_{AR} = 3.8 \times 10^{-43}$ m$^6$/s is the Auger recombination coefficient [35]. The first term on the right-hand side of Eq. (4) describes the carrier transport due to diffusion where $\mu_e = e/(m^*v)$ is the carrier mobility in the CB, and $k_B$ is the Boltzmann constant. Avalanche ionization has been neglected here.
Table 1. Material parameters used in numerical simulations of femtosecond laser-irradiated silicon (wavelength $\lambda = 800$ nm, pulse duration $\tau = 150$ fs).

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band gap energy</td>
<td>$E_g$</td>
<td>1.12</td>
<td>eV</td>
<td>[30]</td>
</tr>
<tr>
<td>Dielectric constant of crystalline silicon</td>
<td>$\varepsilon_{Si}$</td>
<td>13.64 + 0.048i</td>
<td>–</td>
<td>[29]</td>
</tr>
<tr>
<td>One-photon absorption coefficient</td>
<td>$\sigma_1$</td>
<td>$1.021 \times 10^5$</td>
<td>m$^{-1}$</td>
<td>[29]</td>
</tr>
<tr>
<td>Two-photon absorption coefficient</td>
<td>$\sigma_2$</td>
<td>$6.8 \times 10^{-11}$</td>
<td>m/W</td>
<td>[36]</td>
</tr>
<tr>
<td>Carrier collision time</td>
<td>$\gamma$</td>
<td>$1.1 \times 10^{-35}$</td>
<td>s</td>
<td>[27]</td>
</tr>
<tr>
<td>Effective optical mass</td>
<td>$m_e^*$</td>
<td>$1.64 \times 10^{-31}$</td>
<td>kg</td>
<td>[27]</td>
</tr>
<tr>
<td>Auger recombination rate</td>
<td>$C_{AR}$</td>
<td>$3.8 \times 10^{-43}$</td>
<td>m$^8$/s</td>
<td>[35]</td>
</tr>
<tr>
<td>Minimum Auger recombination time</td>
<td>$\tau_{AR}$</td>
<td>$6.0 \times 10^{-12}$</td>
<td>s</td>
<td>[34]</td>
</tr>
<tr>
<td>Minimum electron-phonon coupling time</td>
<td>$\tau_{e\gamma}$</td>
<td>$240 \times 10^{-15}$</td>
<td>s</td>
<td>[33]</td>
</tr>
<tr>
<td>Threshold density for electron-phonon coupling</td>
<td>$N_{th}$</td>
<td>$6.02 \times 10^{26}$</td>
<td>m$^{-3}$</td>
<td>[33]</td>
</tr>
</tbody>
</table>

The intensity in the sample is calculated by numerically solving the equation

$$\frac{dI}{dz} = - (\sigma_1 I + \sigma_2 I^2).$$

(5)

The intensity at the sample surface is $I(t, z = 0) = [1 - R] I_0(t)$, where $R$ is the surface reflectivity at $\lambda = 800$ nm described in the previous Section 2.1 and the incident laser intensity $I_0$ is given by the temporal double-fs-pulse profile via

$$I_0(t) = \frac{F_0}{\tau} \sqrt{\frac{4 \ln 2}{\pi}} e^{-\frac{1}{2} \left(\frac{t - t_1}{\tau}\right)^2} + e^{-\frac{1}{2} \left(\frac{t - t_2}{\tau}\right)^2}. \quad (6)$$

In this superposition of two individual temporally Gaussian pulses of duration $\tau$ (full width at half maximum, FWHM), $F_0$ denotes the peak fluence of each individual pulse of the double-pulse sequence. $t_{1,2}$ denotes the time when the two individual pulse maxima are reached. Then $\Delta t = t_2 - t_1$ defines the double-pulse delay. Note that the first pulse is centered at $t_1 = 0$ fs. Here, $\sigma_\tau$ is given by $\tau / (2 \sqrt{2 \ln 2})$. For the numerical calculations, a pulse duration $\tau = 150$ fs was selected to match the experimental conditions given in Ref. [22].

2.3. Energy relaxation

The free-carrier dynamics is additionally affected by carrier diffusion and energy relaxation, which both strongly depend on the temperature. To calculate temperatures of the free-carriers $T_e$ and the silicon lattice $T_{Si}$, a two-temperature model was used. It is assumed here that CB electrons and VB holes both have the same temperature, i.e., $T_e$. A partial differential equation was used to describe the temperature distribution of the free-carriers given by $[25, 30]$

$$C_e \frac{\partial T_e}{\partial t} = \nabla (\kappa_e \nabla T_e) - \gamma (T_e - T_{Si}) + Q_e,$$

(7)

where $\kappa_e = 2 k_B \mu N_e T_e / e$ is the free-carrier thermal conductivity. The specific heat capacity of the free-carriers is given by $C_e = \frac{3}{2} k_B N_e$. The energy coupling rate between free-carriers and lattice is $\gamma = \frac{C_e}{\tau_{e\gamma}}$, with $\tau_{e\gamma} = \tau_{e\gamma}^0 \left[ 1 + \left( \frac{N_e}{N_e^0} \right)^2 \right]$, $N_{th} = 6.02 \times 10^{20}$ cm$^{-3}$ and $\tau_{e\gamma}^0 = 240$ fs $[25, 33]$. According to recent experiments, $\tau_{e\gamma}$ has been limited to 2 ps at high carrier densities $[37]$.
The laser source term $Q_e = \left[ (\hbar \omega - E_g) \frac{\sigma_1}{\hbar \omega} + (2\hbar \omega - E_g) \frac{\sigma_2}{\hbar \omega} \right] + E_g R_e - \frac{3}{2} k_B T_e \frac{\partial N_e}{\partial t}$ taken from [25] considers the one- and two-photon absorption of the laser beam, the release of energy ($E_g R_e$) due to Auger recombination (band gap energy $E_g = 1.12$ eV), and the last term accounts for the $N_e$-dependent change of the electronic heat capacity. Free-carrier heating has been neglected.

The lattice temperature $T_{Si}$ is calculated by solving the partial differential equation [30]:

$$C_{Si} \frac{\partial T_{Si}}{\partial t} = \nabla (\kappa_{Si} \nabla T_{Si}) + \gamma(T_e - T_{Si}).$$

The second term on the right hand side of Eqs. (7) and (8) considers energy transfer between the free-carriers and the lattice via electron-phonon coupling. The heat capacity of silicon $C_{Si}$ and thermal conductivity $\kappa_{Si}$ both depend on the liquid density fraction $\eta$ in the laser-excited volume and were calculated here via $C_{Si}(T_{Si}) = (1 - \eta) C_{Si}^{(s)}(T_{Si}) + \eta C_{Si}^{(l)}(T_{Si})$ and $\kappa_{Si}(T_{Si}) = (1 - \eta) \kappa_{Si}^{(s)}(T_{Si}) + \eta \kappa_{Si}^{(l)}(T_{Si})$. For non-molten (solid) silicon, the heat capacity $C_{Si}^{(s)}$ and the thermal conductivity $\kappa_{Si}^{(s)}$ are given by $C_{Si}^{(s)} [J/(m^3-K)] = 1.978 \times 10^6 + 3.54 \times 10^3 T_{Si} - 3.68 \times 10^6 T_{Si}^{-2}$ and $\kappa_{Si}^{(s)} [W/(m-K)] = 1.585 \times 10^5 T_{Si}^{-1.23}$, respectively [35]. For the liquid state, the corresponding parameters are $C_{Si}^{(l)}(T_{Si}) = 2.633 \times 10^6$ $J/(m^3-K)$ [38] and $\kappa_{Si}^{(l)}(T_{Si}) [W/(m-K)] = 50.2 + 29.3 \times 10^{-3} (T_{Si} - T_m)$ [39]. The melting temperature $T_m$ of silicon is 1687 K at ambient pressure [40]. Melting is considered in the model of laser heating here by keeping the local temperature constant at $T_m$ until the internal energy increase reaches the melting enthalpy $\Delta H_m = 4.2 \times 10^9$ $J/m^3$ [41].

The boundary conditions for free-carrier transport and thermal energy diffusion were implemented so that free-carriers and energy do not leave the sample of 400 µm thickness. The initial temperature of the system was set to 300 K. The numerical code for solving the partial differential Eqs. (4), (7), and (8) was developed by using the method of finite volumes [42]. The calculations were performed in a one-dimensional approach in the direction normal to the sample surface. The time integration was performed using an implicit scheme. Equation (5) has been solved using a Runge-Kutta 4th-order algorithm [43]. The numerical time-step $\Delta t$ has been decreased until convergence of the solutions was reached ($\Delta t = 10^{-16}$ s). A non-uniform mesh size was used with smaller cells close to the surface. For simplified cases, the validity of the numerical code has been verified with analytical solutions and energy balance monitoring.

2.4. Quantification of the SPP active area

The numerical calculations were performed for different laser fluences $F_0$ up to 0.7 J/cm². For each fluence, the delay $\Delta t$ between the two $\tau = 150$ fs pulses was systematically varied between 0 and 3.5 ps.

For quantification of the SPP active area, the following four steps were conducted.

1. For each laser fluence $F_0$, and double-pulse-delay $\Delta t$, the free-carrier density $N_e$ was calculated as a function of time (up to $t = 5$ ps). Since the proposed mechanism of LSFL formation involves the excitation of SPP and their interference with the incident laser beam [13], the carrier density $N_e^{max}(F_0, \Delta t)$ was identified just at the end of the second femtosecond laser pulse, i.e., at the time $t_{max}$ when its intensity dropped to a fraction $\zeta$ of its maximum value $I_0(t_{max}) = \zeta \times \max\{I_0(t)\} = \zeta \times F_0/\tau \times \sqrt{4 \ln 2/\pi}$. As the interference between the laser and the SPP electromagnetic field should be remarkable, the value of $\zeta$ is expected to be close to unity.

2. As a Gaussian beam with radius $w_0$ having a spatial fluence profile of $F(r) = F_0 e^{-2(\pi r^2/w_0^2)}$
was used, the relation between radial coordinate \( r \) and the local fluence \( F \) is given by

\[
 r(F) = w_0 \sqrt{\frac{1}{2} \ln \left( \frac{F_0}{F} \right)} \quad (9)
\]

3. In the next step, for each fluence \( F \), the radial coordinate \( r(F) \) was calculated according to Eq. (9), and associated with the corresponding value \( N_{e}^{\text{max}}(F, \Delta t) \). In this way, a curve \( N_{e}^{\text{max}}(r(F), \Delta t) \) was constructed.

4. Then, the implicit equation

\[
 N_{e}^{\text{max}}(r(F), \Delta t) = N_{th}^{\text{SPP}} \quad (10)
\]

was solved, where \( N_{th}^{\text{SPP}} \) is the threshold carrier-density necessary for SPP excitation. The value of \( N_{th}^{\text{SPP}} \) is directly calculated from Eq. (2) using the criterion \( \Re \left[ \varepsilon_{e}^{\ast} \left( N_{th}^{\text{SPP}} \right) \right] = -1 \), which can be written as [16]

\[
 N_{th}^{\text{SPP}} = \frac{m_{e}^{2} \varepsilon_{0} \left( \Re \left[ \varepsilon_{Si} \right] + 1 \right)}{e^{2}} \left( \omega^{2} + \nu^{2} \right) \quad (11)
\]

Using the parameters introduced in Section 2.1, \( N_{th}^{\text{SPP}} = 5.27 \times 10^{21} \text{ cm}^{-3} \) is found. With that value, the solution of Eq. (10) provides the SPP excited radius \( r_{\text{SPP}}(\Delta t) \) as a function of the double-pulse delay \( \Delta t \). Finally, assuming a circular shape of the SPP active area, \( A_{\text{SPP}} \) is calculated via

\[
 A_{\text{SPP}}(\Delta t) = \pi r_{\text{SPP}}^{2}(\Delta t). \quad (12)
\]

3. Results and discussion

In this section, the results of the numerical calculations based on the model given in the previous Section 2 are presented. The free-carrier density \( N_{e} \) is calculated as a function of time \( t \), laser fluence \( F_{0} \), and double-pulse delay \( \Delta t \). First, the effect of a double-pulse irradiation on the free-carrier density at the surface is analyzed, before the SPP active area is quantitatively evaluated as function of the double-pulse delay \( \Delta t \). Finally, assuming a circular shape of the SPP active area, \( A_{\text{SPP}} \) is calculated via

\[
 N_{e}^{\text{max}}(\Delta t) = \pi r_{\text{SPP}}^{2}(\Delta t). \quad (12)
\]

Figure 2 exemplifies the results of the numerical calculations of the free-carrier density \( N_{e} \) at the silicon surface upon irradiation with double-pulses of \( \Delta t = 0.5 \text{ ps} \) delay and with three different fluences \( F_{0} = 0.2, 0.3, \) and \( 0.5 \text{ J/cm}^{2} \). The upper graph visualizes the temporal evolution of \( N_{e} \) (left ordinate) on short timescales up to 1 ps, while the lower graph displays the carrier dynamics at longer timescales up to 5 ps. In both graphs the temporal intensity profile of the double-fs-pulse sequence is added for comparison (right ordinate). As seen in Fig. 2(a), the maximum carrier density is reached during the second laser pulse and increases with \( F_{0} \). For the lowest fluence of \( F_{0} = 0.2 \text{ J/cm}^{2} \), the threshold density \( N_{th}^{\text{SPP}} \) is not reached, while for \( F_{0} = 0.3 \text{ J/cm}^{2} \) it is exceeded during the second pulse. For \( F_{0} = 0.5 \text{ J/cm}^{2} \), it is passed already during the first laser pulse. Moreover, the decrease rate (\( \partial N_{e} / \partial t \)) of the carrier density between the two individual femtosecond laser pulses is faster at higher fluences \( F_{0} \). This is attributed to Auger recombination and thermal carrier diffusion which are more efficient at larger carrier densities (recombination) and larger electron temperatures (diffusion). The two mechanisms also account for the carrier density decrease after the second laser pulse, as seen at large times in Fig. 2(b). For all selected fluences and at the given delay, the second pulse increases the carrier density \( N_{e} \) above the level generated by the first one.

Similar calculations have been performed by systematically varying the laser fluence \( F_{0} \) (0.15 J/cm² - 0.70 J/cm²) and the double-pulse delay \( \Delta t \) (0 ps - 3.5 ps). Figure 3 visualizes all the results by plotting the maximum of the free-carrier density \( N_{e}^{\text{max}} \) as a function of both parameters.
Fig. 2. Evolution of the free-carrier density (left ordinates) at the surface of silicon irradiated by a femtosecond double-pulse with a delay of $\Delta t = 0.5$ ps and single pulse fluence of $F_0 = 0.2$, $0.3$ and $0.5$ J/cm$^2$ (pulse duration $\tau = 150$ fs, laser wavelength $\lambda = 800$ nm). (a) Short timescales up to 1 ps. (b) Long timescales up to 5 ps. For comparison, the temporal double-pulse intensity profile is shown (right ordinates). The horizontal lines indicate the threshold density $N_{\text{SPP}}^{\text{th}}$ for SPP excitation. The maximum values of the carrier density $N_{e}^{\text{max}}$ are marked in (b).

in a false-colour map. The solid black isoline indicates where the carrier density equals the SPP threshold density $N_{\text{SPP}}^{\text{th}}$. Above $F_0 = 0.17$ J/cm$^2$, this threshold can be exceeded.

Figure 4 represents a vertical cross section at $F_0 = 0.25$ J/cm$^2$ through the data previously shown in Fig. 3. Two regimes can be distinguished in the evolution of the maximum carrier density with the delay, i.e., a fast decrease within less than 0.15 ps and a slow decrease with a time constant of approximately 8 ps (see the red solid curve). A variation of the model parameters reveals that the fast decay is caused by the nonlinear (two-photon) absorption in the silicon. As the two-photon absorption scales quadratically with the laser beam intensity (see Eq. (5)), it increases the number of laser-generated carriers especially for small delays when the two 150-fs laser pulses are temporally overlapping. Moreover, individually disabling the physical processes confirms that the slow area decrease originates from both the thermal diffusion of carriers and from Auger recombination. The two additional (dashed green and dotted blue) curves represent the values of $N_{e}^{\text{max}}$ calculated according to step 1 in Section 2.4 for two very different intensity-decay levels $\zeta$ = 50\% and 99\%. While the curve of $N_{e}^{\text{max}}$ for $\zeta$ = 50\% almost coincides with $N_{e}^{\text{max}}$, the curve for $\zeta$ = 99\% decays faster with $\Delta t$. This demonstrates
Fig. 3. Maximum free-carrier density as a function of laser fluence $F_0$ and double-pulse delay $\Delta t$. The black isoline represents the SPP threshold density $N_{SPP}^{th}$. The three crosses indicate the parameters used in Fig. 2. (Irradiation parameters: $\tau = 150$ fs, $\lambda = 800$ nm). The vertical dashed line corresponds to the data displayed in Fig. 4.

Fig. 4. Maximum free-carrier density $N_{e}^{max}$ as a function of the double-pulse delay $\Delta t$ (red solid line). The dashed blue and dotted green represent $N_{e}^{max}$ calculated for $\zeta = 50\%$ and $99\%$, respectively. Model parameters: $F_0 = 0.25$ J/cm$^2$, $\tau = 150$ fs, $\lambda = 800$ nm. The dotted horizontal line indicates the threshold density $N_{SPP}^{th}$ for SPP excitation.

that $\zeta$ in the temporal interference criterion (see step 1 in Section 2.4) is important here and should be systematically varied in the comparison of the SPP active area $\Delta_{SPP}$ and the LSFL rippled area $\Delta_{LSFL}$.

Following the complete procedure (steps 1 – 4) described in Section 2.4, the SPP active area has been calculated for the different values of $\zeta = 50\%, 90\%$ and $99\%$. Figure 5(a) visualizes the calculated results of $\Delta_{SPP}$ as a function of the double-pulse delay $\Delta t$ (red curves) along with the experimental data reported in Ref. [22] for the LSFL rippled area after irradiation of silicon by a sequence of 15 double-pulses, each at a total peak fluence of $F_{0,\text{tot}} = 2 \times F_0 = 0.3$ J/cm$^2$ (blue data points). Figure 5(b) shows three representative scanning electron micrographs of the LSFL rippled areas obtained under these conditions for $\Delta t = 0$ ps, 0.7 ps, and 2 ps.
Fig. 5. Calculated SPP active area $A_{SPP}$ as a function of the double-pulse delay $\Delta t$ for three different values of $\zeta = 50\%$, 90\% and 99\% (a). The blue data points of the LSFL rippled area $A_{LSFL}$ are taken from [22] for comparison. Model parameters: $F_0 = 0.40 \text{ J/cm}^2$, $\tau = 150\text{ fs}$, $\lambda = 800\text{ nm}$, $w_0 = 16.5\mu\text{m}$. In (b), representative scanning electron micrographs for delays up to 2 ps are shown. The LSFL rippled area is marked at $\Delta t = 0.7\text{ ps}$.

Quantitative agreement between $A_{SPP}$ and $A_{LSFL}$ is obtained for $\zeta = 99\%$. It must be noted here that the optimum agreement with the experimental data was obtained for $F_0 = 0.40 \text{ J/cm}^2$ which is more than two times larger than the single-pulse fluence $F_0 = 0.15 \text{ J/cm}^2$ used experimentally.

In order to explain this discrepancy between the experimental and theoretical fluence values, some additional aspects must be considered. The numerical calculations performed here assume the interaction of a single double-fs-pulse with a geometrically flat surface of single-crystalline silicon. However, in the experiments, multiple double-pulse sequences were required to form the LSFL at the surface resulting in a periodically corrugated surface. Typically, a few incubating pulses are needed to reach the ablation condition and to generate a rough surface [44]. Additional laser pulses then form the LSFL via scattering and SPP excitation and further reinforce the grating-like surface morphology via inter-pulse feedback [14, 24].

Upon irradiation with multiple laser pulses, the maximum number of laser-induced carriers generated at the surface will be additionally affected by several inter- and intra-pulse effects – even if the laser peak fluence ($F_{0,\text{tot}}$) is kept constant during the pulse irradiation sequence:

1. **Incubation effects (inter-double-pulse):** Due to incubation effects, the damage threshold fluence of the material $F_{th}$ decreases with increasing number of (single) laser pulses [30]. For Ti:sapphire 130-fs laser pulse irradiation of silicon, this effect was studied already in detail in [10]. For less than $N = 1000$ laser pulses, an incubation law of the form $F_{th}(N) = F_{th}(1) \times N^{\xi - 1}$ was found, with the single-pulse damage threshold $F_{th}(1) = 0.26 \text{ J/cm}^2$ and an incubation coefficient $\xi = 0.84$ [10]. As a result, for a fixed fluence $F_0$, the ratio $F_0/F_{th}$ increases, resulting in a larger number of laser-generated carriers at the silicon surface. Hence, for $N = 15$ laser pulses as used in [22], the threshold ratio $F_{th}(1)/F_{th}(15)$ is $\sim 1.5$. Considering this effect along with the somewhat different pulse duration ($\tau = 150\text{ fs}$), the numerical calculations indicate that the experimental value of $F_0 = 0.15 \text{ J/cm}^2$ used in [22] for the multiple pulse irradiation is equivalent to the
irradiation by an increased effective single-pulse fluence of 0.27 J/cm².

2. Geometrical surface modifications (inter-double-pulse): Once the initially flat silicon surface becomes corrugated due to ablation after a certain number of double-pulse sequences, the next laser pulse will significantly change its interaction (absorption) due to geometrical effects. As in the vicinity of sharp features of the surface topography a significant enhancement of the electromagnetic field (by factors up to 10) can occur [19, 45, 46], the spatial carrier density distribution may become inhomogeneous and increases locally at a rough or rippled surface. Along with the increased optical absorption, also the sample reflectivity decreases due to the geometrical corrugation from initially $R_{\text{flat}} = 0.33$ for a flat silicon surface at normal incidence ($\lambda = 800$ nm) down to values below $R_{\text{corr}} < 0.1$ [47]. However, even for a perfectly absorbing surface ($R_{\text{corr}} = 0$) the latter effect can increase the absorbed laser pulse energy by up to 50% $[(1-R_{\text{corr}})/(1-R_{\text{flat}}) = (1-0)/(1-0.33) = 1.5]$. 

3. Structural surface modifications (inter-double-pulse): Upon laser pulse irradiation in the ablative regime, also a residual melt layer is left at the surface which resolidifies after each double-pulse impact. For (single-pulse) fluences below 0.58 J/cm², a superficial amorphous layer of some tens of nanometer thickness is formed at the surface which exhibits a surface reflectivity increased by up to a few percent and which has a 25-times increased linear absorption coefficient compared to crystalline silicon [48]. This inter-pulse effect additionally may increase the number of laser-induced carriers, as due to the amorphization, the optical absorption occurs in a reduced energy deposition depth. Moreover, laser-generated structural effects in the silicon lattice can lead to trapping of carriers by defect states on the picosecond timescale [49].

4. Phase transitions (intra-double-pulse): If the delay between the first and the second pulse of the double-pulse sequence is large enough, thermal phase transitions will occur if the fluence of the first pulse is sufficiently high. Thermal melting of semiconductors was shown to occur on the timescale of a few picoseconds [50] while ablation starts typically after tens to hundreds of picoseconds [51]. This effect can contribute to deviations between the experimentally observed LSFL area and the calculated SPP active area. Above a certain number of double-pulse sequences, the silicon melts already after the first pulse. The optical surface properties then turn within a few picoseconds to that of metal with a few nanometer optical penetration depth only acting during the second laser pulse. As the energy deposition depth is simultaneously decreased, the energy of the second pulse is deposited in a layer excited by the first pulse already, which later undergoes almost complete ablation and is, therefore, not contributing to the LSFL topography. In this way, the effect may shield the pulse energy of the second pulse when melting occurs between the two fs-laser pulses.

The theoretical description of these intra- and inter-pulse effects in multiple irradiation sequences is beyond the scope of this work. However, the discussed effects justify that the fluence $F_0$ was varied systematically in the numerical calculations to obtain the best match between the SPP active area and the LSFL rippled area.

4. Conclusions

The laser-induced carrier dynamics in the conduction band of silicon upon irradiation with femtosecond double-pulse sequences of varying delays up to $\sim 4$ ps was numerically calculated using the two-temperature model, considering carrier generation via one- and two-photon absorption, carrier diffusion and Auger recombination effects as well as their impact on the
transient optical properties. At sufficiently high laser fluences, the semiconducting material transiently turns to a metallic state allowing the excitation of surface plasmon polaritons. Additionally taking into consideration a spatially Gaussian beam profile along with an interference criterion between the laser and the plasmon electromagnetic fields, the SPP active area was quantified as a function of the double-pulse delay and compared to experimental data of the LSFL rippled area observed in recent experiments. Both the SPP active area and the LSFL area show two characteristic decay-times – one of $\tau_1 \sim 0.147$ ps and another one of $\tau_2 \sim 13.0$ ps. Quantitative agreement between both the experiments and calculations was obtained when multi-pulse driven inter- and intra-pulse feedback effects are considered. Our results directly support the SPP-based LSFL formation mechanism proposed earlier for silicon and identify the nonlinear (two-photon) absorption to be responsible for the rapid area decrease ($\tau_1$) while carrier diffusion and Auger recombination effects account for the slower area decrease ($\tau_2$). The temporally resolved calculations of the carrier dynamics manifest that the intra-pulse energy deposition step is responsible for the spatial characteristics of the rippled surface which is later reinforced through inter-pulse feedback.

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