The Generic Inverse Problem in Differential Galois Theory

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Abstract. Polynomial Galois theory studies extensions of fields generated by solutions of polynomial equations. In a similar way, one can consider differential fields, i.e., fields endowed with a derivation operation, and study the extensions of such fields generated by solutions of linear homogeneous differential equations. The subfield of constants of the base field, that is, the subfield of elements whose derivative is zero plays a very important role in this theory.

In the polynomial situation the group of automorphisms of a Galois extension over the base field is finite. In the differential case this group has the structure of a linear algebraic group over the constants and a differential Galois extension with a given differential Galois group will be the function field of a principal homogeneous space for the group.

The generic inverse problem asks, given a linear algebraic group G defined over an algebraically closed field of constants C whether there are a differential field $\mathcal{F}$ and a differential Galois extension $\mathcal{E} \supset \mathcal{F}$ with differential Galois group G, whose generators satisfy universal relations in the sense that every differential Galois extension of a differential field with field of constants C, whose differential Galois group is G, has generators satisfying at least those relations.

In this talk I will first introduce the basics and then explain my recent work (joint with Arne Ledet) solving the generic inverse problem for various groups.