

# Motzkin's density problem and the equivalent colouring problems in Graph Theory

Abstract

For a given set  $M$  of positive integers, Motzkin asked to find the quantity  $\mu(M)$ , which is the supremum of all upper densities  $\bar{\delta}(S)$ , where  $S$  is a set of nonnegative integers with  $a \in S, b \in S \implies a - b \notin M$ . This problem is equivalent to two colouring problems in Graph Theory. The first problem is the "Asymptotic T-colouring Efficiency" due to Rabinowitz and Proulx and the second problem is the "Fractional chromatic number of the distance graph generated by  $M$ " due to Chang, Liu and Zhu. In this seminar I will survey the progress in the density problem and show the equivalence of the density problem with the colouring problems.