Control of mechanical systems subject to impacts

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Summary

- Overview of control problems for systems subject to impacts
- Description of the class of mechanical systems considered (non-smooth impacts)
- Preliminaries: PD control laws from the observed state
- A parameterized family of stabilizing compensators
- An illustrative example
- An extension
- Conclusions
Control problems in presence of impacts

- Problems for which “classical” solutions do still work (possibly, weakening the requirements)
  - Stabilization of contact configurations through PD controllers from state measurements
  - Tracking of trajectories involving infinite impacts through PD controllers from state measurements

- Problems solved by “ad hoc” techniques
  - Observers
    - of velocity
      - full order
      - reduced order
    - of state
  - Parameterization of stabilizing compensators for a contact configuration (“Youla-Kucera”-like) from position measurements

  Solution: the observer takes into account the impacts, since jumps are imposed to the estimated state variables at the impact times.
Control problems in presence of impacts (2)

- Problems solvable only thanks to the presence of impacts
  - Control of (otherwise) uncontrollable systems (= Jugglers)
    - systems with linear unconstrained dynamics (linear in absence of impacts)
  - State observers for (otherwise) unobservable systems
    - linear systems: solutions valid in general cases
    - nonlinear systems: examples
The class of considered systems

In absence of impacts and contacts:

\[ T \ddot{q}(t) + U_2 q(t) + u_1(t) = \tau(t), \quad \forall t \geq t_0, \]

\( q(t) \in \mathbb{R}^n \) is the vector of the Lagrangian coordinates,
\( T \in \mathbb{R}^{n \times n} \) is the symmetric and positive definite inertia matrix,
\( U_2 \in \mathbb{R}^{n \times n} \) is symmetric,
\( u_1(t) \in \mathbb{R}^n \) are known external forces (e.g., the gravity force)
\( \tau(t) \in \mathbb{R}^n \) is the vector of the control forces.

Linear inequality constraint:

\[ J \dot{q}(t) \leq 0, \quad \forall t \geq t_0, \]

\( J' \in \mathbb{R}^n, \ J \neq 0. \)
The class of considered systems (2)

Simplifying assumption: the impacts are perfectly elastic.

A time $\hat{t} \in \mathbb{R}, \hat{t} > t_0$, is an impact time if $Jq(\hat{t}) = 0$ and $J\dot{q}(\hat{t}^-) > 0$.

$T = \{t_i, \ i \in \mathbb{N}\}$ denotes the set of the impact times (ordered so that $t_{i+1} > t_i$)

For $t \in (t_i, t_{i+1}), \ i \in \mathbb{Z}^+$: $T\ddot{q}(t) + U_2q(t) + u_1(t) = \tau(t) + R(t)$, where $R(t)$ is the reaction force:

$$R(t) = \begin{cases} 
0, & \text{if } (Jq(t) < 0) \text{ or } (Jq(t) = 0 \text{ and } c(t) \leq 0), \\
\frac{c(t)}{JT^{-1}J'}, & \text{if } Jq(t) = 0 \text{ and } c(t) > 0,
\end{cases}$$

$c(t) := JT^{-1}(\tau(t) - U_2q(t) - u_1(t))$.

$R(t)$ can be different from zero only during permanent contacts.
The class of considered systems (3)

Restitution rule: \( \dot{q}(t_i^+) = Z \dot{q}(t_i^-) \), where \( Z := I_n - \frac{2}{J_T} J' J \).

Set of admissible initial conditions:

\[ \mathcal{A} := \left\{ [q' \ v']' \in \mathbb{R}^{2n} : (J q \leq 0) \text{ and } (J v \leq 0 \text{ if } J q = 0) \right\}. \]

Overall dynamic system:

\[
S \begin{cases}
    \dot{q}(t) &= v(t), \quad t \geq t_0, \quad t \notin T, \\
    T \dot{v}(t) &= -U_2 q(t) - u_1(t) + \tau(t) + R(t), \quad t \geq t_0, \quad t \notin T, \\
    \dot{q}(t_i^+) &= Z \dot{q}(t_i^-), \quad t_i \in T, \\
    q(t_i^+) &= q(t_i^-), \quad t_i \in T, \\
    y &= q(t), \quad t \geq t_0
\end{cases}
\]

where \([q'(t_0) \ v'(t_0)]' \in \mathcal{A}\).
Preliminaries

Let $W \in \mathbb{R}^{n \times n}$ be such that:

$$W' T W = I,$$
$$W' J' J W = \text{diag}(J T^{-1} J', 0, \ldots, 0).$$

To compute $W$:

- factorize $T$ so that $T = H H'$;
- compute $w_1 := \frac{H^{-1} J'}{\|H^{-1} J'\|};$
- complete $w_1$ to a orthonormal basis $\{w_1, w_2, \ldots, w_n\}$ of $\mathbb{R}^n$;
- let $W := (H^{-1})' [w_1 \ldots w_n]$. 
Preliminaries: PD control laws from the observed state

\begin{align*}
\dot{q}(t) &= \dot{v}(t) + K_1 (y(t) - \hat{q}(t)), \quad t \geq t_0, \quad t \notin T, \\
T \dot{v}(t) &= -K_p \hat{q}(t) - K_v \dot{v}(t) + r(t) + R(t) + K_2 (y(t) - \hat{q}(t)), \quad t \geq t_0, \quad t \notin T, \\
\tau(t) &= U_2 y(t) + u_1(t) - K_p \hat{q}(t) - K_v \dot{v}(t) + r(t), \quad t \geq t_0, \quad t \notin T, \\
\hat{q}(t_i^+) &= Z \hat{q}(t_i^-), \quad t_i \in T, \\
\dot{v}(t_i^+) &= Z \dot{v}(t_i^-), \quad t_i \in T,
\end{align*}

\begin{align*}
K_p &:= (W^{-1})' \text{diag}(k_{p,1}, \ldots, k_{p,n}) W^{-1}, \\
K_v &:= (W^{-1})' \text{diag}(k_{v,1}, \ldots, k_{v,n}) W^{-1}, \\
K_1 &:= W \text{diag}(k_{1,1}, \ldots, k_{1,n}) W^{-1}, \\
K_2 &:= (W^{-1})' \text{diag}(k_{2,1}, \ldots, k_{2,n}) W^{-1},
\end{align*}

\[k_{p,h}, k_{v,h}, k_{j,h} > 0, \forall j \in \{1, 2\}, \forall h \in \{1, \ldots, n\}.\]
Rewrite the closed-loop system using the state vector \( x_{so}(t) = \begin{bmatrix} q'(t) & v'(t) & \tilde{q}'(t) & \tilde{v}'(t) \end{bmatrix}' \), where \( \tilde{q} = q - \hat{q} \) and \( \tilde{v} = v - \hat{v} \):

\[
\begin{align*}
\dot{q}(t) &= v(t), \quad t \geq t_0, \quad t \notin T, \\
T \dot{v}(t) &= -K_p (q(t) - \tilde{q}(t)) - K_v (v(t) - \tilde{v}(t)) + r(t) + R(t), \quad t \geq t_0, \quad t \notin T, \\
\dot{\tilde{q}}(t) &= \tilde{v}(t) - K_1 \tilde{q}(t), \quad t \geq t_0, \quad t \notin T, \\
T \dot{\tilde{v}}(t) &= -K_2 \tilde{q}(t), \quad t \geq t_0, \quad t \notin T, \\
v(t_i^+) &= Z v(t_i^-), \quad t_i \in T, \\
\tilde{q}(t_i^+) &= Z \tilde{q}(t_i^-), \quad t_i \in T, \\
\tilde{v}(t_i^+) &= Z \tilde{v}(t_i^-), \quad t_i \in T.
\end{align*}
\]

**Assumption 1** Let \( x_{so}(t_0) \in \mathcal{A} \times \mathbb{R}^{2n} \) and \( r(\cdot) \in \overline{C}^0([t_0, +\infty)) \) be such that there are no finite accumulation points of the impact times.
Theorem 1 [1] Under Assumption \(\Box\)

(i) the equilibrium \(x_{so} = 0\) is **globally exponentially stable**, i.e., there exist \(\alpha > 0\) and \(\beta > 0\) such that, for \(r(\cdot) = 0\):

\[
\|x_{so}(t)\| \leq \alpha \exp(-\beta(t - t_0)) \|x_{so}(t_0)\|, \quad \forall t \geq t_0;
\]

(ii) the closed-loop system is globally bounded-input bounded-state (shortly, **BIBS** or **ISS**) stable, i.e., there exist two functions \(\gamma_x(\cdot, \cdot)\), \(\gamma_r(\cdot)\) such that, if \(\|r(t)\| \leq r_M, r_M \in \mathbb{R}^+\), for all \(t \geq t_0\), then:

\[
\|x_{so}(t)\| \leq \gamma_x(\|x_{so}(t_0)\|, t - t_0) + \gamma_r(r_M), \quad \forall t \geq t_0.
\]

\(\gamma_x(\cdot, \cdot) : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+\) and \(\gamma_r(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+\), both strictly increasing with respect to the first argument, \(\gamma_x(s, \cdot)\) decreasing, with \(\lim_{z \to +\infty} \gamma_x(s, z) = 0\) for every \(s > 0\), and such that \(\gamma_x(0, z) = 0\), for all \(z \in \mathbb{R}^+\) and \(\gamma_r(0) = 0\).
Hints of the proof of Theorem 1

In the variables $z = W^{-1} q$, $v_z = W^{-1} v$, $\tilde{z} = W^{-1} \tilde{q}$ and $\tilde{v}_z = W^{-1} \tilde{v}$, the closed-loop system becomes:

\[
\begin{align*}
\dot{z}(t) &= v_z(t), \quad t \geq t_0, \quad t \notin T, \\
\dot{v}_z(t) &= -\text{diag}(k_{p,1}, \ldots, k_{p,n})(z(t) - \tilde{z}(t)) \\
&\quad -\text{diag}(k_{v,1}, \ldots, k_{v,n})(v_z(t) - \tilde{v}_z(t)) + r_z(t) + R_z(t), \quad t \geq t_0, \quad t \notin T, \\
\dot{\tilde{z}}(t) &= \tilde{v}_z(t) - \text{diag}(k_{1,1}, \ldots, k_{1,n})\tilde{z}(t), \quad t \geq t_0, \quad t \notin T, \\
\dot{\tilde{v}}_z(t) &= -\text{diag}(k_{1,1}, \ldots, k_{1,n})\tilde{z}(t), \quad t \geq t_0, \quad t \notin T, \\
v_z(t_i^+) &= \text{diag}(-1, 1, \ldots, 1)v_z(t_i^-), \quad t_i \in T, \\
\tilde{z}(t_i^+) &= \text{diag}(-1, 1, \ldots, 1)\tilde{z}(t_i^-), \quad t_i \in T, \\
\tilde{v}_z(t_i^+) &= \text{diag}(-1, 1, \ldots, 1)\tilde{v}_z(t_i^-), \quad t_i \in T.
\end{align*}
\]
Hints of the proof of Theorem 1 (2)

By reordering the state variables in the new state vector

\[ x_{zo} = [\tilde{z}_1 \; v_{z,1} \; \tilde{v}_1 \; \tilde{z}_n \; \tilde{v}_{z,n} \; z_n \; v_{z,n}]' \]

it follows that \[ \frac{1}{\sigma(W)} \| x_{so} \| \leq \| x_{zo} \| \leq \frac{1}{\sigma(W)} \| x_{so} \| . \]

Let \( x_z := [\tilde{z}_1 \; v_{z,1} \; \ldots \; \tilde{z}_n \; v_{z,n}]' \), \( \tilde{x}_o := [\tilde{z}_1 \; \tilde{v}_z, 1 \; \ldots \; \tilde{z}_n \; \tilde{v}_z, n]' \),

\[ A_z := \text{blockdiag} \left( \begin{bmatrix} 0 & 1 \\ -k_{p,1} & -k_{v,1} \end{bmatrix}, \ldots, \begin{bmatrix} 0 & 1 \\ -k_{p,n} & -k_{v,n} \end{bmatrix} \right), \]

\[ A_o := \text{blockdiag} \left( \begin{bmatrix} -k_{1,1} & 1 \\ -k_{2,1} & 0 \end{bmatrix}, \ldots, \begin{bmatrix} -k_{1,n} & 1 \\ -k_{2,n} & 0 \end{bmatrix} \right). \]
Hints of the proof of Theorem 1 (3)

Consider the solutions $P_z$ and $P_o$ of the two Liapunov equations:

$$P_z A_z + A_z^T P_z = -I_{2n}, \quad P_o A_o + A_o^T P_o = -I_{2n},$$

and define

$$V_z = x_z^T P_z x_z, \quad V_o = \tilde{x}_o^T P_o \tilde{x}_o,$$

and the following candidate Liapunov function:

$$V_{zo} = V_z + \varepsilon V_o,$$

which for convenience is rewritten as

$$V_{zo} = x_{zo}^T P_{zo} x_{zo},$$

with

$$P_{zo} = \text{blockdiag}(P_{zo,1}, \ldots, P_{zo,n}),$$

$$P_{zo,h} = \text{blockdiag}(P_{z,h}, \varepsilon P_{o,h}), \quad h = 1, \ldots, n.$$
Hints of the proof of Theorem 1 (4)

Let $A_{zo}$ be the dynamic matrix of the closed-loop system written with state vector $x_{zo}$ with $R_z(t)$ set equal to zero, and define

$$Q_{zo} = -\left(P_{zo} A_{zo} + A_{zo}^T P_{zo}\right),$$

which is positive definite if and only if $\varepsilon > \varepsilon^*$, with $\varepsilon^* := \max_{h\in\{1, ..., n\}} \{f(k_p, h, k_v, h)\}$.

If $r(\cdot) = 0$, for $t \in (t_i, t_{i+1})$, we have:

$$\dot{V}_{zo}(t) = \begin{cases} 
-x_{zo}^T(t) Q_{zo} x_{zo}(t), & \text{if } R_z(t) = 0, \\
-x_{zo}^T(t) \overline{Q}_{zo} x_{zo}(t), & \text{if } R_z(t) \neq 0,
\end{cases}$$

where $\overline{Q}_{zo}$ is equal to $Q_{zo}$ apart from its $4 \times 4$ upper left block, which is equal to $\text{diag}(1, 1, \varepsilon, \varepsilon)$: same diagonal elements of $Q_{zo}$ but no off-diagonal elements.
Hints of the proof of Theorem 1 (5)

$V_{zo}$ is continuous at the impact times (it can be easily checked in view of the block diagonal form of $P_{zo}$).

Let $\xi = \min\{1, \varepsilon, \lambda_m(Q_{zo})\}$.

$\Rightarrow$ ITEM (i) (exponential stability):

$$\|x_{so}(t)\| \leq \frac{\sigma(W)}{\sigma(W)} \sqrt{\frac{\lambda_M(P_{zo})}{\lambda_m(P_{zo})}} \|x_{so}(t_0)\| \exp \left( -\frac{\xi}{2 \lambda_M(P_{zo})} (t - t_0) \right),$$

$\Rightarrow$ ITEM (ii) (BIBS stability):

$$\|x_{zo}(t)\| \leq \sqrt{\frac{\lambda_M(P_{zo})}{\lambda_m(P_{zo})}} \|x_{zo}(t_0)\| \exp \left( -\frac{\xi}{2 \lambda_M(P_{zo})} (t - t_0) \right) + \frac{2 \lambda^2_M(P_{zo})}{\lambda_m(P_{zo})} \xi \|W\| r_M,$$
The proposed family of stabilizing compensators

\[ \tau(t) = U_2 y(t) + u_1(t) - K_p \hat{q}(t) - K_v \hat{v}(t) + y_Q(t), \quad t \geq t_0, \quad t \notin T, \]

\[ \dot{\hat{q}}(t) = \hat{v}(t) + K_1 (y(t) - \hat{q}(t) - r(t)), \quad t \geq t_0, \quad t \notin T, \]

\[ T \dot{\hat{v}}(t) = -K_p \hat{q}(t) - K_v \hat{v}(t) + y_Q(t) + R(t) + K_2 (y(t) - \hat{q}(t) - r(t)), \]

\[ \quad t \geq t_0, \quad t \notin T, \]

\[ \dot{x}_Q(t) = A_Q x_Q(t) + B_Q (y(t) - \hat{q}(t) - r(t)), \quad t \geq t_0, \quad t \notin T, \]

\[ y_Q(t) = C_Q x_Q(t) + D_Q (y(t) - \hat{q}(t) - r(t)), \quad t \geq t_0, \quad t \notin T, \]

\[ \hat{q}(t_i^+) = Z \hat{q}(t_i^+), \quad t_i \in T, \]

\[ \hat{v}(t_i^+) = Z \hat{v}(t_i^+), \quad t_i \in T, \]

\[ x_Q(t_i^+) = x_Q(t_i^-), \quad t_i \in T, \]

where \( y_Q(t) \in \mathbb{R}^n, x_Q(t) \in \mathbb{R}^m, A_Q \in \mathbb{R}^{m \times m} \) has all the eigenvalues with negative real part and the triple \((A_Q, B_Q, C_Q)\) is reachable and observable.
Main theorem

Denote by \( x_{cc}(t) = \begin{bmatrix} q^T(t) & v^T(t) & \tilde{q}^T(t) & \tilde{v}^T(t) & x_Q^T \end{bmatrix}^T \) the state of the closed-loop system.

**Assumption 2** Let \( x_{cc}(t_0) \in A \times \mathbb{R}^{2n+m} \) and \( r(\cdot) \in \overline{C}^0([t_0, +\infty)) \) be such that there are no finite accumulation points of the impact times for the closed-loop system.

**Theorem 2** [2] Under Assumption 2

(i) the equilibrium \( x_{cc} = 0 \) is globally exponentially stable;

(ii) the closed-loop system is globally bounded-input bounded-state stable.
Hints of the proof of Theorem 2

Consider the state vector

\[ x_{zc} = \begin{bmatrix} x_z \\ \tilde{x}_o \\ x_Q \end{bmatrix} \]

\( \leftarrow \) positions and velocities in the "z" frame
\( \leftarrow \) estimation errors in the "z" frame
\( \leftarrow \) state of subsystem ”Q”

and the solution \( P_Q \) of the Liapunov equation

\[ P_Q A_Q + A_Q^T P_Q = -I_m, \]

and define

\[ V_z = x_z^T P_z x_z, \quad V_o = \tilde{x}_o^T P_o \tilde{x}_o, \quad \text{as before} \]

and the following candidate Liapunov function:

\[ V_{zo} = V_z + \varepsilon V_o + \eta x_Q^T P_Q x_Q. \]
Hints of the proof of Theorem 2 (2)

Let

- $\mathbf{A}_{zc}$ be the dynamic matrix of the closed-loop system with state vector $\mathbf{x}_{zc}$ with $\mathbf{R}_z(t) \equiv \mathbf{0}$,
- $\mathbf{A}_{zc}$ the same matrix for when $\mathbf{R}_z(t) \neq \mathbf{0}$,
- $Q_{zc} = - \left( \mathbf{P}_{zc} \mathbf{A}_{zc} + \mathbf{A}_{zc}^T \mathbf{P}_{zc} \right)$,
- $\overline{Q}_{zc} = - \left( \mathbf{P}_{zc} \mathbf{A}_{zc} + \mathbf{A}_{zc}^T \mathbf{P}_{zc} \right) = Q_{zc} - \text{some off-diagonal terms}$.

\[
Q_{zc} = \begin{bmatrix}
\mathbf{I}_{2n} & Q_1 & Q_2 \\
Q_1^T & \varepsilon \mathbf{I}_{2n} & \eta Q_3 \\
Q_2^T & \eta Q_3^T & \eta \mathbf{I}_m
\end{bmatrix}.
\]
Hints of the proof of Theorem 2 (3)

Letting $\chi_h = \|Q_h\|$, $h = 1, 2, 3$, if $r(\cdot) = 0$, for $t \in (t_i, t_{i+1})$, we have

$$\dot{V}_{zc} \leq - \begin{bmatrix} \|x_z\| & \|\tilde{x}_o\| & \|x_Q\| \end{bmatrix} \hat{Q}_{zc} \begin{bmatrix} \|x_z\| \\ \|\tilde{x}_o\| \\ \|x_Q\| \end{bmatrix}, \quad \hat{Q}_{zc} = \begin{bmatrix} 1 & -\chi_1 & -\chi_2 \\ -\chi_1 & \varepsilon & -\eta \chi_3 \\ -\chi_2 & -\eta \chi_3 & \eta \end{bmatrix}.$$

Leading principal minors of $\hat{Q}_{zc}$:

$$\{1, \varepsilon - \chi_1^2, \varepsilon \eta - \eta^2 \chi_3^2 - \eta \chi_1^2 - 2 \eta \chi_1 \chi_2 \chi_3 - \varepsilon \chi_2^2\}.$$

The choices

- $\eta = 2 \chi_2^2$ ( $(C_Q, A_Q)$ observable $\Rightarrow C_Q \neq 0 \Rightarrow \chi_2 > 0$),
- $\varepsilon > \varepsilon^*$, with $\varepsilon^* := 4 \chi_3^2 \chi_2^2 + 2 \chi_1^2 + 4 \chi_1 \chi_2 \chi_3$,

render $\hat{Q}_{zc}$ positive definite.
An illustrative example

2-degrees-of-freedom system:

- **system description**:

  \[
  \ddot{q}_1(t) = \frac{\tau_1(t)}{M}, \quad t \notin T, \\
  \ddot{q}_2(t) = \frac{\tau_2(t)}{M} - g, \quad t \notin T, \\
  \dot{q}_1(t_i^+) = \dot{q}_2(t_i^-), \quad t_i \in T, \\
  \dot{q}_2(t_i^+) = \dot{q}_1(t_i^-), \quad t_i \in T,
  \]

- **control goal**: position regulation to the origin;

- **uncertainty**: \( M \) is unknown, the nominal value is \( M_0 \).
An illustrative example (2)

PD control from the observed state (no “Q” parameter):

- nominal and perturbed systems have a stable equilibrium position;
- regulation is achieved only in the nominal case;
- “coupling” effect the constraint: if $M > M_0$ the vertical “disturbance” affects also the horizontal position;
An illustrative example (3)

Output feedback controller with parameter “Q”:

- A suitable choice of “Q” allowed to implement an controller based on the internal model principle for the unconstrained system;
- Regulation is achieved also in the perturbed cases.
An extension

Assume for simplicity that there are no permanent contacts.

Consider the family $\mathcal{K}$ of stabilizing compensators that have jumps both at the impact times $t_i \in T$ and at jump times $\theta_i \in \Theta$ and satisfy the following conditions.

- **Continuous-time description:**

  \[
  \begin{align*}
  \dot{x}_K(t) &= A_K x_K(t) + B_K y(t), \quad t \notin T \cup \Theta, \\
  u(t) &= C_K x_K(t) + D_K y(t), \quad t \notin T \cup \Theta, \\
  \tau(t) &= U_2 y(t) + u_1(t) + u(t), \quad t \notin T \cup \Theta;
  \end{align*}
  \]

- the **jump times** $\theta_i$ are determined by a set of $\mu_K$ constraints of the following kind:

  \[
  F^K_i x_K(t) \leq 0, \quad \forall t \geq t_0, \; i = 1, \ldots, \mu_K;
  \]
An extension (2)

- at the **impact times**:
  \[ x_K(t_i^+) = H_0^K x_K(t_i^-), \quad t_i \in T, \ t_i \notin \Theta \]

- at a **“simple” jump time** due to the \( j \)-th constraint:
  \[ x_K(\hat{\theta}_i^+) = H_j^K x_K(\hat{\theta}_i^-); \]

- at a **“multiple” jump time** due to the constraints \( \{j_1, \ldots, j_\nu\} \):
  \[
  x_K(\hat{\theta}_i^+) = \begin{cases} 
  H_{j_1}^K \cdots H_{j_\nu}^K x_K(\hat{\theta}_i^-), & \text{if } \theta_i \notin T, \\
  H_0^K H_{j_1}^K \cdots H_{j_\nu}^K x_K(\hat{\theta}_i^-), & \text{if } \theta_i \in T;
  \end{cases}
  \]
An extension (3)

- suitable conditions on $F^K_1, \ldots, F^K_{\mu_K}, H^K_0, \ldots, H^K_{\mu_K}$, to ensure that
  - the jump times do not depend on the compensator input,
  - matrices $H^K_i$ pairwise commute

- a quadratic Lyapunov function $V_c(x_c) = x'_c P_{c,K} x_c$ which is continuous
  - both at the impact times $t_i \in T$ and at the jump times $\theta_i \in \Theta$ exists for the closed-loop system.

The family $\mathcal{K}$ of compensators satisfying the above conditions can be parameterized by a family $\mathcal{K}_Q$ of compensators $K_Q$, similar in spirit to the family proposed here, with the stronger property:

**Theorem 3** For each stabilizing compensator $K^* \in \mathcal{K}$, there exists $Q^* \in Q$ such that the closed-loop system corresponding to $K_{Q^*}$ is able to reproduce, if correctly initialized, every trajectory of the closed-loop system corresponding to $K^*$. 

Conclusions

• The problem of **stabilizing a contact configuration** in presence of **non-smooth impacts** has been considered.

• A **family of stabilizing compensators** based on a free parameter “Q” has been proposed, whose state is subject to jumps at the impact times.

• Its efficacy has been proven by **Liapunov techniques**.

• The usefulness of a free parameter for obtaining additional control requirements has been shown by an **example** in which **robust regulation** has been achieved.

• An **extension** of the proposed family of compensators has been outlined, in which the state of the compensator is allowed to jump also at **“autonomously generated” jump times**.
 Related work by the authors
